III. Knowledge

1. Sensation and Argument

We acquire knowledge partly through sensory perception and partly through reflection. From time immemorial, sensation, sight in particular, has been regarded as the prototype of knowledge acquisition. We acquire knowledge by keeping our eyes open and absorbing the world through them. If we were to close our eyes or lose our sight, we would acquire less knowledge.

But what kind of knowledge do we acquire through our eyes? Do we see "mere sense data" – red spots, for example – in our field of vision? No. We perceive "sense data" as *something*, as we already realised when we were hearing voices and reading texts. If, for example, we see a red spot, we may be looking at a wine stain on a table cloth; if we hear a whistle in the mountains, it may be the whistle of a marmot; if we smell an odour, it may be that of a cigar; if we taste something sour, it may be lemon juice; if we feel a cold object in the dark, we may decide that it is a key. The same shape, for example, ⌧, can be seen as an envelope, a pitched roof from above, or a roof truss from below.

Looking at human beings, too, as a rule, we perceive not merely bodies, but men, women, children, bank clerks, workers, asylum seekers, "the motley crew of humanity" (Wilhelm Busch). The French novelist Marcel Proust (1871-1922) writes: "Even the simple act which we describe as 'seeing some one we know' is, to some extent, an intellectual process. We pack the physical outline of the creature we see with all the ideas we have already formed about him, and in the complete picture of him which we compose in our minds those ideas have certainly the principal place."[1] What a person sees depends both on what he is looking at and on "what his previous visual-conceptual experience has taught him to see".[2]

However, it is not only everyday perception, but also scientific perception, that sees something as something. As Thomas Samuel Kuhn (1922-1996) writes in *The Structure of Scientific Revolutions* (1962): "When Aristotle and Galileo looked at swinging stones, the first saw constrained fall, the second a pendulum."[3] It is not possible to build a theory on pure observation even in empirical science. Observation always involves a theory. Observation and theory, so to speak, merge into one. The more we know, the more we see something as something. The more flowers we know, the more we recognise the specificity of individual flowers, for example, the specificity of bluebells. It is not until we analyse these sensory impressions that we can try to distinguish "pure" sense data from their interpretation, even though there may be no sharp dividing line between data and interpretation. The sensation is mediated through the "lenses" of our interpretation. There is no such thing as unmediated sensory knowledge. Unmediated sensory knowledge, like a pure sense datum, is an abstraction.

In fact, sensory perception is a relationship between (a) a perception and (b) a sense datum perceived as (c) something. It is a tripartite relationship. The sense datum can be perceived from two different angles: on the one hand, in its physical or chemical aspect, on the other hand, as a phenomenal fact.

1 A la recherche du temps perdu, Volume 1, Du coté de chez Swann, Part 1, Combray. Transl. C. K. Scott-Moncrieff, Swann's Way, New York 1922.

2 Kuhn, Structure, Chapter 10, 113.

3 Kuhn, ibid. 121.

The sensory datum can therefore be analysed physically or chemically: Lightning, for example, is an electric discharge of short duration and high voltage. But however we analyse the datum, it must make an impact on our sensory organs if it is to be accessible to us at the phenomenal level. The electric discharge makes an impact on our retina. Our eye has a causal relationship with its surroundings and it is through that relationship that it experiences any changes to the retina. According to the causal theory of perception, the causal relationship is *necessary* if we are to have any knowledge involving sensory experience.

Some changes are forwarded to the nervous system and the brain as signals. They generate sensations, in the present instance, a sensation of light. This is then interpreted as something specific, say, as the perception of a flash of lightning. The same applies to hearing, smelling, tasting and touching. For example, we interpret certain sound waves as the solitary song of a blackbird before a thunderstorm. The creative contribution of consciousness is most recognisable in connection with ambiguous shapes such as ⊠ mentioned above.

Sensory knowledge contains a passive and an active part. The passive part is made up of what the body absorbs, the stimulus, and what the stimulus generates, the perception. The active part is what we make of the perception. The decisive factor, according to the causal theory of perception, is that our sensory knowledge is necessarily limited from the outset. We are unable to perceive things that do not affect our senses or exchange any physical energy with them. For example, we can imagine a thunderstorm with our inward eye, and Ludwig van Beethoven (1770-1828) can even make us apprehend one in the fourth movement of his Pastoral Symphony. Nevertheless, while listening to the Pastoral Symphony, we cannot see any lightning with our actual eyes, because there is no visible lightning.

Of course, we can foresee or predict future thunderstorms. Although sensory perception is the prototype of knowledge acquisition, it is not the only form of it. Sensory perception would restrict us to the present and make us unable either to draw conclusions from the past or to arrive at inferences for the future. But even if we are given sensory perception together with the memory of other sensory impressions received, we are still unable to formulate a single scientific law. Moreover, there is knowledge – particularly mathematical and logical – that cannot be gained through sensory perception alone. Therefore, in addition to knowledge acquired through the senses – which depends on our interpretation, to boot – we must assume a further source of knowledge acquired, not through sensory perception, but through reflection.

Reflection makes use of reason. By reason, we mean non-sensory knowledge. It is knowledge gained not through our senses, but through the meaning of words. Reason, in contrast to sensory perception, draws conclusions. Granted, our perception of something as something is also based on conclusions: We see something as something because our past experience has taught us to see something as something. But sensory perception on its own does not draw any conclusions. It is reason that draws conclusions. Conclusions need not be expressly put into words. But if they are, it is done by means of arguments.

An argument in the technical sense consists of sentences that have a certain relationship with each other. This relationship is inferential. The sentences that contain the reasons for an inference are called the premises; the sentence that contains the inference is called the conclusion. Therefore, an argument consists of a premise, or some premises, and a conclusion. Two types of argument are particularly important, the deductive and the inductive.

2. Deductive and Inductive Arguments

Let us consider these two types of argument by way of two elementary examples (the line between the premises and the conclusion stands for "therefore"):

All humans are mortal.
All philosophers are human.
———————————————
All philosophers are mortal.

The following applies to deductive arguments:

a) If all the premises are true, and the inference is drawn according to valid rules, it is necessary that the conclusion also will be true. The conclusion of a valid deductive argument, then, preserves the truth of the premises. In this example, the conclusion "All philosophers are mortal" preserves the truth of the premises "All humans are mortal" and "All philosophers are human".

However, we must make a distinction between the truth of the premises and the conclusion and the validity of the argument. Truth refers either to the premises or to the conclusion; validity refers to the argument that consists of both the premises and the conclusion.

A deductive argument is valid if the affirmation of the premises and the negation of the conclusion result in a logical contradiction between the premises and the conclusion. A logical contradiction is the conjunction of a proposition with the negation of that proposition. For example, a logical contradiction arises if we assert that all humans are mortal and all philosophers are human but not all philosophers are mortal. If all humans are mortal and all philosophers are human, then all philosophers are also mortal. To say that philosophers are both mortal and not mortal – combining affirmation of the premises with negation of the conclusion – is a logical contradiction. Because the affirmation of the premises and the negation of the conclusion results in a contradiction, the argument is therefore valid.

The argument would also be valid if it came to light that not all humans are mortal, but some are immortal, or that not all philosophers are human, but some are non-human. For it would still be a logical contradiction to say that not all philosophers are mortal. Thus, the validity of a deductive argument rests only on the logical relationship between the premises and the conclusion, and not on the truth. Therefore, the following deductive argument is also valid, even though it sets out from an untrue premise and leads to an untrue conclusion:

All humans are immortal.
All philosophers are human.
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All philosophers are immortal.

This argument is valid, although not sound. Only a deductive argument that is valid and has true premises is sound. A deductive argument is unsound if it is not valid or if one or more of its premises are false. So we can distinguish not only between truth and validity (cf. p. 61), but also between truth, validity and soundness.

Naturally, a valid and sound deductive argument need not have two premises. It can have only one. For example, the premise "It is not the case that some humans are not mortal" leads to the conclusion "All humans are mortal."

Only in a valid deductive argument does the conclusion necessarily preserve the truth of the premises. The same does not apply to the conclusion of an invalid deductive argument. In the following example, the conclusion does not preserve the truth of the deductive argument, which has nothing but true premises, but which is nevertheless invalid:

If a philosopher owns all the gold in the vaults of the Bank of England, he is rich
No philosopher owns all the gold in the vaults of the Bank of England.

No philosopher is rich.

A deductive argument, then, can have true premises and still be invalid. A deductive argument is invalid if the affirmation of the premises and the negation of the conclusion do not result in a logical contradiction between the premises and the conclusion. In the above example, there is no logical contradiction if the premises are affirmed and the conclusion negated. The negation of "No philosopher is rich" is "It is not the case that no philosopher is rich." What follows from this is: "Some philosophers are rich." There is no logical contradiction in asserting that although no philosopher owns all the gold in the vaults of the Bank of England, there are some rich philosophers. Some philosophers may be rich for other reasons. That is why the argument is invalid. A deductive argument, then, is either valid or invalid. There is no such thing as a halfway valid deductive argument.

b) The information content of the conclusion is already present, albeit undeveloped, in the premises. The conclusion only unfolds that knowledge. Valid deductive arguments, therefore, unfold existing knowledge. But this does not mean that our own knowledge is not expanded in the process. Thus, the conclusion of the argument

All humans are fallible.
All philosophers are human.

All philosophers are fallible.

contains an insight that some philosophers may not yet possess. We can also be taught something new by deductive conclusions. There is scope for deductive discoveries. It is by no means the case that we have already drawn all the conclusions from all the premises we know. Arthur Schopenhauer (1788-1860) cites the following example:

All diamonds are stones.
All diamonds are combustible.

Therefore some stones are combustible.[4]

This is a fact that we probably did not know before, even though the new knowledge was already present, hidden in the old.

Examples of deductive conclusions are found not only in formal logic, but also in arithmetic and geometry. The best-known example is probably the *Elements* of Euclid (about 325 BC). In this work, propositions are proven on the basis of principles and claims. These propositions are also called theorems, principles are also called axioms, and claims are also called postulates. Axioms and postulates are premises; theorems are conclusions. The method of proof consists in deducing theorems according to certain rules of inference. Euclid does not put these rules into words. But without doubt, by this method we,

[4] Schopenhauer, W II, Book 1, Chapter 10, 118. Transl. Haldane and Kemp.

too, can learn something that we did not know before, at least not in a developed form. Take, for example, the proposition that "in any triangle the sum of any two angles is less than two right angles."[5] This could come as a new insight to most school children.

Frege, too, argues that arithmetical truths are obtained deductively, but can nevertheless increase our knowledge, which should "put an end to the widespread contempt for analytic judgments and to the legend of the sterility of pure logic". Thus, a schoolboy's knowledge will increase as much through the realisation that there are more prime numbers than he has ever been shown, or that "(a+b) × (a–b)" leads to "(a×a) – (b×b)", as it will through the awareness that some stones are combustible. To give another example, our knowledge is broadened by learning that there are some prime numbers with more than 258,716 digits, which used to be regarded as the largest prime number so far calculated.

Deductive conclusions must be distinguished from inductive ones. To show this, I will again choose an elementary example:

All the philosophers observed up to day X have died.

All philosophers are mortal.

This is an example of an inductive argument, to which the following applies:

a) If the premise (or premises) is true, it is not necessary that the conclusion is also true, as there is no valid rule that allows the truth of the premise (or premises) to be transferred to the conclusion. The premise "All the philosophers observed up to day X have died" refers either to a past day or the current

[5] Elements, Book 1, Proposition 17. Transl. Joyce.

one. The conclusion "All philosophers are mortal" includes all future philosophers. However, a day in the future could see the birth of a philosopher who will not die. The conclusion is fallible, because its truth does not follow from that of the premise. An inductive argument, then, is not logically valid, since the affirmation of the premise(s) and the negation of the conclusion do not produce a logical contradiction between the premise(s) and the conclusion. The conclusion of an inductive argument does not preserve the truth of the premises, but expands their content.

Accordingly, the conclusion of a general inductive argument may be wrong, if it is refuted, or falsified, by experience. In fact, no conclusion of a general inductive argument can be true in a strict sense, because no conclusion of a general inductive argument can be proven, or verified, completely. To verify a general inductive argument completely, we would need to be in a position to cite all future examples, that is, a potentially infinite number of them. Not least, we would have to include all future philosophers. In order to do that, not only would we have to be immortal ourselves, but, as I have said, one day a philosopher would have to be born who would never die. The conclusion above is confirmed, without exception and therefore indisputably, only up to the present moment.

Other conclusions reached inductively, for example, that philosophers are hard to understand, are less well confirmed. However, the degree of confirmation is not determined by the meaning of the words – although this must be defined sharply enough – but by experience. An inductive argument is never either valid or not valid, but rather more valid or less valid. But even when it is more valid or less valid according to experience, it is not more or less logically valid but always logically invalid. A conclusion reached inductively can only be more or less well verified, or confirmed.

b) The information content of the conclusion is not found in the premises, as it is, in undeveloped form, in deductive arguments. Inductive conclusions do not disclose what we already know in a hidden form: They project existing knowledge into the future.

Examples of inductive arguments occur in most scientific disciplines. All the natural laws go beyond merely describing the condition of the world to date. Even a simple one, such as Hooke's "The pulling force of an elastic spring is proportional to its extension", projects existing knowledge into the future. That the extension is proportional to the pulling force is valid for all elastic springs, including those in epochs to come. Natural laws are not obtained by merely listing empirical data; generally, though not always, they are articulated on the basis of a working hypothesis. However, they are confirmed only by empirical data available up to the present and therefore fundamentally fallible. All the natural laws that are valid today may no longer be valid tomorrow. By tomorrow, the earth may no longer rotate round on its own axis, and by tomorrow, the sun may not rise again.

Inductive arguments – let me repeat it once more to avoid misunderstandings – are not logically valid. In inductive arguments, the affirmation of the premise(s) and the negation of the conclusion do not produce a logical contradiction.

Despite their logical invalidity, inductive arguments play a more important part in the empirical sciences and in everyday life than deductive ones. We use inductive arguments not only in many empirical sciences, medicine for example, but above all in our daily routine, as shown by the following reflections: Because so far the sun has always risen, it will also rise in future. Because so far fire has always burnt us, it will also burn us in future. Because bread has nourished us till now, it will also nourish us in future. Because the chair we sit on has not floated off into the air by itself so far, it will not cease to obey the laws of gravity in future, etc.

All these conclusions are fallible, but without the instinctive subjective belief in their truth, we would not be able to perform the simplest, most mundane actions. That is why David Hume, in his *An Inquiry Concerning Human Understanding* (1748), described induction – or, to be more precise, custom (cf. p. 70) – as "the great guide of human life".[6] A belief in the "validity" of our inductive arguments is essential to our activity and survival in this world. Conversely, in a world without laws, no predictions or plans would be possible and our expectations would be constantly disappointed. Such a world would be like a nightmare in which we would not be able to take one step securely or eat one meal in peace. Conceivably, what was firm ground yesterday would dissolve under our feet today, the bread that has nourished us would poison us today, and the chair we are sitting on would lift off into the air. Even the most universal laws of nature, such as the principle of conservation, would become void. Our belief in the existence of natural laws would vanish. "There would be an end at once of all action, as well as of the chief part of speculation."[7] Nevertheless, the belief that the laws of yesterday and today will still be valid tomorrow is not, and cannot be, justified by a logically valid argument. Theoretically, tomorrow everything could be completely different.

[6] Hume, Enquiry, Section 5, Part I, 44.
[7] Hume, ibid., 45.

3. How Do We Justify the Conclusion of an Inductive Argument?

Let us assume that a creature capable of reason from a distant planet has come to our earth for a day. It sees that the sun rises, senses that fire burns, feels that bread nourishes, etc. Does it therefore infer that the same will happen in future? Hardly. But if it has spent a week on earth, it will expect the phenomena to repeat themselves. And if the phenomena repeat themselves over a year, or indeed over several years, it will probably conclude that the same phenomena will repeat themselves forever. There is no logical justification for this conclusion. Nevertheless, we all draw it instinctively. A baby already learns from experience: "As soon as he cried he was fed" (Wilhelm Busch).

Even animals harbour such inductive expectations, although they do not formulate them in a language, and it is doubtful that they are able to draw inductive conclusions at a pre-language level. Thus, a cat "expects" that the milk that nourished it in the past will also nourish it in the future. A chicken "expects" that the person who brought it food in the past will continue to feed it. However, as Bertrand Russell (1872-1970) remarks, it can end tragically for the chicken: "The man who has fed the chicken every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken."[8]

On what extra-logical ground do we extend the content of the experiences we have had to experiences we have not yet had? By what extra-logical right do we project our past empirical knowledge into the future? That is the so-called induction problem. David Hume did not discover it, but he was the first to

[8] Russell, Problems, Chapter 6, 98.

recognise its full importance, even though he does not use the term “induction”. He would say: Custom is the principle that enables the transition from what we know to what we do not yet know. In his view, custom plays the decisive part in both the evolution and the justification of these conclusions. Custom is why we make the transition, and why we are allowed to make it. This justification is also called the induction principle.

However, custom as a justification is contradicted by the certainty with which we draw these inductive conclusions. We do not know that tomorrow the sun will rise, fire will burn, bread will nourish again, etc., but our certainty seems justified by the fact that such inductive conclusions – despite the tragic error of Russell’s chicken – are rarely refuted by nature. The chicken has had its neck wrung. But this was because it had developed somewhat undifferentiated ideas about the uniformity of nature rather than about the uniformity of human behaviour. The sun does not set and rise everywhere daily, for example, at the North or the South Pole. But this does not disprove the fact that in our part of the world, so far, it has set and risen every day. If these conclusions could be justified merely by custom, the confidence based apparently on nature would be incomprehensible. Why should nature follow our customs?

Hume’s problem was presented in a new version by Nelson Goodman (1906-1998) in his *Fact, Fiction and Forecast* (1955). While Hume was concerned with justifying our customary inductive inferences, Goodman shows that we need further reasons for our preference of accustomed generalisations over unaccustomed ones. Let us assume that all the emeralds we have seen up to a certain point in time, *t*, are green. And let us call an artificial colour, which is green up to a certain point in time *t*, but red afterwards, “grue”. Our experience up to *t* will support both inductive generalisations, that all emeralds are green and that they are “grue”. As both general hypotheses are equally well confirmed by our experience up to *t*, we can replace "green" with "grue" and, instead of "All emeralds are green", say "All emeralds are grue." But then we are equally entitled to the conclusion that after *t*, all emeralds are green and that after *t*, all emeralds are "grue". Given a certain quantity of data, and using such artificial predicates, we can find a large, indeed potentially infinite, number of inductive generalisations with equal rights. For now, I will select only one.

Why do we not usually draw conclusions that project such artificial predicates into the future, for instance, that all emeralds are "grue"? Goodman's answer is that conclusions that do not use artificial predicates such as "grue" are better embedded in our usage than conclusions that do. That is why we choose one kind rather than the other, and we feel entitled to say that emeralds will continue to be green in future. But this answer is at least as unsatisfactory as Hume's. Why should nature obey our existing linguistic customs?

An apparent way out is to attribute probability to our inductive conclusions, if not truth. According to our empirical observations up to now, it is not true, but very probable, that the same thing will occur again. Here we have to make a distinction between the probability of events and the probability of hypotheses. In the first case, we attribute probability to events, in the second, to hypotheses about events. As hypotheses are formulated in propositions, we can also speak of propositional probability.

In the first case, probability is interpreted as the relative frequency of events in a sequence of events. This is empirical. Thus, it is an empirical fact that lung cancer occurs more frequently among smokers than among non-smokers.

In the second case, probability is understood as a relationship between propositions that partly imply one another. This approach is logical. Therefore, this kind of propositional probability is also called logical probability, although "logical" should rightly be placed between quotation marks. According to this interpretation, the proposition that all emeralds are green partly gives rise to the proposition that they will also be green in future. The proposition that fire has always been known to burn partly suggests that it will also burn in future. The proposition that bread nourished the hungry in the past suggests that it will also nourish them in future, etc. If the propositions about past observations are so well confirmed that the general propositions logically follow from them, we have the extreme case of the probability of the general proposition being equal to one. If, however, the propositions about past observations are so badly confirmed that it is the negation of the general proposition that follows from them, we have the other extreme case of the probability of the general proposition being equal to zero. Between these two extremes, we have a continuum of cases to which the "inductive logic" developed by Carnap applies (1950).

This "inductive logic" is very different from deductive logic, whose arguments are either valid or invalid. It is a logic of probability, whose arguments are more or less valid and whose conclusions are more or less probable. To quantify the "more" or the "less", the probabilities are allocated numbers between one and zero. Thus, it may be found that the probability of bread nourishing, based on past empirical observations, amounts to 0.999999. Therefore, the past propositions would imply a general hypothesis that "bread nourishes" to a degree of 0.999999.

But, to justify such a probability inference, we would need a legitimate reason for drawing conclusions concerning future experiences from past ones. We would need an altered induction principle which would make conclusions concerning the future, drawn from past experiences, probable, albeit not logically valid. How can we justify this inductive probability principle? Perhaps because it has been true in the past? This would throw us back to the question of why it should also be true in future. To answer that, we would need a probability principle of a higher order making it probable that the probability principles to date will also be probable in future, and so on to infinity.

But let us assume that we can measure the probability of a general hypothesis without such a probability principle. In that case, we might prefer the well-confirmed general hypothesis H_1 to the badly confirmed hypothesis H_2 if the probability of H_1 is greater than that of H_2. The probability of H_1 is greater than that of H_2 if the past propositions imply hypothesis H_1 to a higher degree than H_2. Both H_1 and H_2 are general hypotheses. General hypotheses, like laws of nature, apply, by definition, to an infinite number of future cases. Therefore, an infinite number of cases to which H_1 and H_2 could apply are as yet unconfirmed. But since all the cases confirmed in the past amount only to a finite number, both H_1 and H_2 would have the same degree of probability – that is, zero.

If we deduct a finite number of confirmed cases from an infinite number of unconfirmed ones, the difference between the finite numbers of confirmed cases will be the same, that is, zero. Infinity minus however small or however large a finite number still amounts to infinity. Thus, *"in an infinite universe* (it may be infinite with respect to the number of distinguishable things, or of spatio-temporal regions), *the probability of any (non-tautological) universal law will be zero."*[9] But our universe may continue to exist for an infinitely long time. What we have so far observed is only an infinitesimal part of the universe. Therefore, inductive logic does not supply a good reason to character-

[9] Popper, LSD, New Appendix, Section 7, 313. Italics in the original. Transl. Popper et al.

ise the well-confirmed general hypothesis H_1 as more probable than the badly confirmed H_2.

Nevertheless, we might subjectively regard hypothesis H_1 as more probable than H_2. We might underline this subjective probability by being prepared to bet on H_1 rather than H_2. Of course, we are only prepared to bet on single events, and not on any general hypotheses with an infinite number of unconfirmed cases. Only events can be dated; general hypotheses cannot. A "rational gambler" would take the objective chances into account in order to win his bet. However, faced with an infinity of unconfirmed events, nobody who makes a bet can win it. Thus, even in the case of rational gamblers prepared to bet, the interpretation of subjective probability fails to supply a logical reason for regarding the general hypothesis H_1 as more probable than H_2.[10]

That is why Karl Popper (1902-1994), in *The Logic of Scientific Discovery* (1959; original version *Logik der Forschung*, 1934), chose a different route. He argues that empirical laws are neither completely verifiable nor probable. At the same time, they can be refuted, or falsified, by a single counter-example. For instance, the proposition "All ravens are black" can be refuted by the existence of a single white raven, unless we believe that blackness is an essential characteristic of a raven and therefore do not call a white raven a raven in the first place. But the white raven I once saw in the Negev Desert was called a raven. If, then, empirical laws are neither completely verifiable nor probable, we may still adhere to them, so long as they are not falsified by a contradictory experience. Now, our usual empirical laws – for example, that the sun rises, fire burns and bread

[10] For further information, see Popper, LSD, New Appendix, Section 9, Communication 3, 359-373, Subsection 11, 368. Transl. Popper et al.

nourishes – are not falsified as a rule. As they have not been falsified, they have been corroborated. An empirical law or a system of empirical laws, that is, a theory, is deemed to have been corroborated if it has been proved true by experience. Since the empirical laws mentioned have stood the test of time, we can obey them.

What is right about this reflection is that empirical laws are not completely verifiable, but can be falsified by a single counter-example, even if any counter-example is hypothetical. The above-mentioned white raven could have been an albino or fallen into a bag of flour or been painted white a short while earlier. We must therefore make a distinction between falsifiability as a logical possibility and falsifiability as an actual decision, and indicate precisely what would constitute a counter-instance. The empirical law "All ravens are black" is falsified by the existence of a white raven only if we actually define the bird in question as both a raven and white.

But Popper denies empirical laws any validity by his clear admission that Hume has posed a problem that cannot be solved by deductive logic. Popper did not find a positive solution to Hume's problem either, but he isolated a part of the original problem and proposed a negative solution for it: The conclusions of inductive arguments are not completely verifiable, but they can be falsified by a single counter-example. But Popper's negative answer does not solve the original problem – "What is our extra-logical justification for projecting our past knowledge into the future?" – by supplying a logical reason. There is no logical reason to project our past knowledge into the future just because it has been corroborated. Indeed, Hume's problem cannot be solved by logical deduction. Inductive conclusions do not acquire any validity through definition, as do deductive ones.

Popper's positive answer – that unfalsified conclusions have been corroborated – turns the original problem of what extra-logical justification we may have for projecting our past knowledge into the future into a test by time. But why should any empirical laws that have been corroborated till now also be corroborated in the future? That is exactly what we do not know, and shall never know. Therefore, I believe that Hume, in spite of Popper's attempt, is right in principle when he says: "It is not reasoning which engages us to suppose the past resembling the future."[11]

4. The Induction Principle as a Hypothetical Postulate of Practical Reason

With the concept of corroboration, Popper brings a new point of view into play – cognitive valuation. If a law of nature has been corroborated, it is *worth* accepting. But in the process, he moves in principle from the ambit of theoretical reason to that of practical reason, whereas Hume, in the passage quoted above,[12] has theoretical reason in mind. Let us pursue this point of view further. We want to accept Popper's critique and grant the laws of nature neither truth nor probability. Nevertheless, we can allow them a kind of extra-logical justification, that is, a justification not by theoretical but by practical reason.

So far, we have considered only theoretical reason. However, there is also a practical reason, since we obviously draw

[11] I owe this hypothesis to Feyerabend, Probleme des Empirismus, Chapter 14, 362. English version by Feyerabend.

[12] Practical reason, in Hume's view, is only an imprecise and unphilosophical figure of speech for something that does not exist in reality. With this, he departs from both common and philosophical usage. Cf. Treatise, Book 2, Section 3, 413-416.

not only theoretical conclusions but also practical ones. Theoretical reason infers what will be from what was or what is; practical reason, on the other hand, infers what one ought to do. The empirical laws that have been corroborated express the knowledge acquired by humanity to date. This knowledge has clearly proved to be an advantage in the struggle for survival. Conversely, it would be a great disadvantage not to know what we know from experience, even though not everyone would have wished to put it in the words of Willard Van Orman Quine (1908-2000): "Creatures inveterately wrong in their inductions have a pathetic but praiseworthy tendency to die before reproducing their kind."[13]

The survival value of the past experience of humankind is my starting point. From those past experiences that have been *corroborated*, we can deduce directions for our actions which should also be valid in the future. Because the past experience that fire burns has stood the test of time, it is expedient to assume that it will continue to do so, and we would be well advised not to put our hands in the flames, if it can be avoided. Because bread nourished us in the past, it is expedient to assume that it will also nourish us in the future, etc. Therefore, instead of understanding the induction principle as a principle that tells us what is, I understand it as a norm that tells us what to assume and what to do on the basis of the assumptions that have been corroborated. The justification of this norm is not that I attribute any truth or probability to it, but that I see an advantage in following it. If, then, an inductive conclusion is not logi-

[13] Quine, Ontological Relativity, Chapter 5, Natural Kinds, 126. For such a pragmatic justification of induction, see Reichenbach, Probability, 469-482, and Salmon, 1991, 99-122. I reserve this justification for hypotheses that have been corroborated.

cally valid, it is, as a rule, advantageous. A ban on induction would amount to an invitation to suicide. It is, for example, expedient, or indeed imperative, to assume that for some time to come, fire will continue to burn, bread to nourish, etc. If we assumed that fire no longer burns, or bread no longer nourishes, we would burn ourselves or starve to death, as the case may be.

Of course, the survival value of our inductive generalisations need not be as obvious as that. But if we were to assume, for example, that in future ravens will be white and emeralds "grue", that stones will fly up in the air instead of falling down, that the planets will no longer revolve in ellipses, etc., we would be able to continue living, but sooner or later we would find ourselves at a disadvantage in comparison to those who draw the more "valid", that is, more expedient, conclusions. Since the empirical laws cohere among themselves, we cannot abandon some without abandoning others. That is why usually not one empirical law has been corroborated, but a whole system of them. The pillars of the system, again, are some basic laws, such as the principle of conservation. It is expedient to assume that such a system that has been corroborated will be preserved in future, even if not every single law is important for our survival.

To that extent, an inductive conclusion – embedded in such a system – is not logically valid, but neither is it irrational. The alternative of assuming no inductive principle would surely be more irrational. Likewise, with our survival in mind, it would be more irrational to assume a principle whereby the opposite of our past experiences will occur. However, we are not dealing here with a valid conclusion of theoretical reason, but with a postulate of practical reason. This postulate is justified by the fact that, as a rule, it is expedient for our survival in a wide sense, even though once in a while it may not be so in exceptional cases.

Thus, Popper reports an episode of ergot poisoning in a French village.[14] Here the assumption that bread, or corn, nourishes was not borne out. But this experience does not force us to doubt the general law that has otherwise been well corroborated. The ergot poisoning is an example of how a general hypothesis has been falsified as a logical possibility, but is upheld, nevertheless, because we do not posit the counter-example as a criterion of the falsification of the whole law. After all, it could transpire that the cause of the disaster was not the ergot but the poisoned soil. Despite this mishap, it is more expedient to assume that bread nourishes than that it poisons.

Inductive validity, therefore, is not a question of either/or, but a matter of degree, since there are also degrees of expediency. Thus, it will be more expedient in the near future to prefer an empirical law that has been well corroborated – say, "The pulling force of an elastic spring is proportional to its extension" (Hooke) – to one that has been corroborated less well. These degrees of expediency could be quantified, in analogy to the degrees of inductive probability, as degrees of rational eligibility. If the past propositions have been corroborated so well that the corresponding law logically follows from them, we have the extreme case of the degree of rational eligibility being equal to one. If, on the other hand, the past propositions have been corroborated so badly that what logically follows from them is the negation of a corresponding law, we have the other extreme of the degree of rational eligibility being equal to zero.

Between these two extremes, we would again have a continuum of cases subject to the logic of preference.[15] This is nei-

[14] Popper, Objective Knowledge, Chapter I, Section 6.

[15] For such logic, cf. Henrik von Wright, Logic of Preference, esp. §1-8, 7-20. It does not seem to have been applied to the problem of induction. Cf., e.g.

ther a deductive logic, whose arguments are either logically valid or invalid, nor an inductive logic, whose arguments are more or less (theoretically) valid and whose conclusions are quantifiably more or less probable. It would be a purposive logic, whose arguments are more or less (practically) valid and whose conclusions are more or less expedient in the sense of maximising more or less the expected utility. To quantify that "more" or that "less", we could allocate to the degrees of rational eligibility numbers between zero and one, but only for the finite range. That way, we would be able to establish, for example, that in a finite future, the degree of rational eligibility of the empirical law whereby bread nourishes will be equal to 0.999999, that is, nearly one.

Therefore, while the induction principle is not a principle of theoretical reason, it is, in my view, a natural and legitimate postulate of practical reason. For conclusions that have been corroborated well or even as completely consistent, it only makes explicit what we tacitly or implicitly expect, that is, that the future will be uniform with the present. In that sense, the induction principle, too, is an *institution* we tacitly accept.

An institution is a systematic framework which normatively stabilises our actions, for the future, as it has done before. The induction principle, understood normatively, seems to be our justification for projecting our past knowledge into the future. It arises from an urge that is too strong to be suppressed without running the risk of endangering our own survival and that of the human species. In this sense, the institution of induction really plays the part of the "great guide of human life" (Hume). Like a

Popper's disciple Watkins, Science and Scepticism, Epilogue. For the present state of the problem, cf. John Vickers, "The Problem of Induction", *The Stanford Encyclopedia of Philosophy* (Fall 2014 edition).

guide, it tells us what to do. Like a guide, it issues to human beings the order: "If you want to survive and stay healthy, you should assume that, given laws that have been corroborated, the future is uniform with the past." Such an order is a conditional or hypothetical imperative. It remains one, even if the order is misleadingly clothed in the form of an absolute or categorical proposition describing the future.

Of course, here, too, it could be asked: Why should nature obey our demand for uniformity? The answer would be: because this demand itself is "natural" in so far as it was always obeyed by the empirical laws of nature that have been corroborated. But just because nature obeyed this demand in the past, why should it also do so in future? To this question there is no theoretical answer, and there will never be one, because we cannot foresee the future of nature with any certainty – and because things can turn out differently from our expectations.

Therefore, Popper is right in principle in saying: "I do not know – I only guess",[16] even though he is deviating from everyday usage, which allows us sometimes to talk about knowing when we are merely guessing. But it is equally right that knowing, here, lays no claim whatsoever to theoretical infallibility. We do not need any theoretical infallibility. John Stuart Mill (1806-1873) aptly stated this: "There is no such thing as absolute certainty, but there is assurance sufficient for the purposes of human life. We may, and must, assume our opinion to be true for the guidance of our own conduct."[17]

Certainty and assurance are different things, even though this distinction is hardly ever made in everyday life. Certainty is something psychological, assurance something practical. We do

[16] Popper, Conjectures, Chapter XI, 317.
[17] Mill, Liberty, Chapter 2, 81.

not know whether fire will still burn tomorrow, but we make sure that it will not burn us tomorrow. For the purposes of human life, we usually do not need more than this practical assurance based on a rational – that is, here, expedient – choice. This practical assurance is probably the foundation of our certainty – our belief – that our past inductive conclusions will continue to be valid in future. In a theoretical respect, however, all inductive conclusions retain an irreducible remnant of irrationality. Theoretically, tomorrow everything could in fact be different. But it is not merely a custom, but also a command of practical reason, to assume that this will not be the case. In this sense, induction is really “the great guide of human life”.

5. When Are Axioms True?

But even valid deductive arguments need not always lead to true conclusions (cf. p. 62). A conclusion must be true only if all the premises of a deductive argument are true and the argument valid. But when are the premises of a deductive argument true? The premises of a deductive argument are considered undoubtedly true only if they are first premises. First premises are also called axioms. Their truth seems to be timeless and ubiquitous, “without, however, being provable by a chain of logical inferences” (Frege).[18] But what is the criterion of the truth of an axiom? A criterion is a necessary and sufficient condition of something, comparable to a litmus test.

Let us take the ninth axiom of Euclid’s *Elements* as an example: “The whole is greater than the part.” People think – as scientists and philosophers have done for 2,000 years – that it is the *evidence* that makes this proposition true. The word “evi-

[18] Frege, Foundations of Geometry, 262. Transl. Kluge, 273.

dent" literally means "plain to see". What "catches the eye", what is clear and obvious, is evident. As little as I doubt that it is bright outside when the sun shines in a cloudless sky, as little do I doubt that the whole is greater than the part. Both notions make immediate sense, one to my eyes, the other to my reason. Trying to prove something that is evident – to vary a saying attributed to Aristotle – is like trying to prove with a candle that it is bright when the sun shines.

The axioms in Euclid's *Elements*, as he formulates them, refer only to finite figures. But how about infinite figures? With infinite figures, is the whole still greater than the part? If the part has an infinite number of elements, how can the whole be even greater than the part? In fact, if we follow the definition of infinite sets provided by Georg Cantor (1845-1918) in *Contributions to the Founding of the Theory of Transfinite Numbers* (1915), we find that the axiom in question is valid in one sense and not valid in another: "Every transfinite set T has subsets T_1 which are equivalent to it."[19] A transfinite set is an infinite set. Cantor's definition, then, asserts at one and the same time that the whole of the set is greater than its parts, and that it is not. A glance at the following illustration will explain the apparent contradiction. Remember that the line is supposed to consist of an infinite number of points:

So, on the one hand, the whole of the line from A to B is longer than the part from A to C. On the other hand, the quantity of the points in the partial line AC is equal to that in the whole line AB, since both are infinite. Therefore, in Cantor's

[19] Cantor, Contributions, Part III, Chapter 9, §6, 295. Transl. Jourdain. Part III missing.

terminology, the whole and the part are "equipollent", or to use a more familiar term, "equivalent". However, it may not be immediately recognised how a partial set can be equal in its extent to the complete set. I must therefore establish that fact by means of a definition.

The definition of parallelism was also assumed to supply a true proposition, which was called the parallel axiom. Euclid defines parallel as follows: "Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction."[20] This definition was used by others to formulate the parallel axiom, which is not found in explicit form among the nine axioms of Euclid.[21] According to the parallel axiom, for every plane in which there is a straight line G and a point P that does not lie on G, there is one straight line G' that goes through this point P and that is parallel to the straight line G.

This seemed so plausible that, to my knowledge, nobody seriously doubted its truth before the 19th century. The argument was about whether it was a first geometrical premise (that is, a geometrical axiom) or only a conclusion (that is, a theorem). There were many attempts to prove the parallel axiom, that is, to derive it from the other axioms of Euclid's system of axioms. But these attempts were all circular and therefore faulty. A

[20] Elements, Book 1, Definition 23. Transl. Joyce.

[21] Instead, Euclid uses the 5th postulate to prove the axiom that we know today as a parallel axiom: "If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles" (Elements, Book 1, Postulate 5, Transl. Joyce). For an intelligible presentation of the problem, see Bonola, Non-Euclidean Geometry, 1-8. Transl. Carslaw.

proof is circular if the truth of the conclusion is already assumed in the truth of the premises.

In 1816, however, the mathematician Carl Friedrich Gauss (1777-1855) proved that the parallel axiom could not be derived from the other axioms. This raised the question of whether it was possible to do without it. Gauss answered this question in the affirmative, and he constructed a consistent geometry without a parallel axiom, which, however, he did not dare to publish. Later (in 1832) János Bolyai (1802-1860) and Nikolai Ivanovich Lobachevsky (1792-1856) also proved that the parallel axiom cannot be derived from the other axioms. They therefore felt justified – independently of Gauss and of each other – in constructing geometries in which the parallel axiom was no longer included. A little later still (in 1854), Bernhard Riemann (1826-1866) constructed a geometry with more than one parallel through a point P. In fact, it was found that there were infinitely many parallel lines.

In the geometry of Euclid, then, we have one straight line G' that goes through the point P and is parallel to the straight line G; in the geometries of Bolyai and Lobachevsky, we have no straight line G'; and in the geometry of Riemann, we have more than one straight line G'. All this is no longer immediately plausible or evident.[22] In other words, mere evidence can give a valuable hint about the truth of axioms such as the ninth or the parallel, but we cannot always rely on evidence alone where axioms are concerned. Evidence serves only at first sight as a criterion of the truth of axioms. A criterion at first sight is a prima facie criterion. And a prima facie criterion can be invalidated by more accurate reflection.

22 For an intelligible presentation of these non-Euclidean geometries, see Bonola, Non-Euclidean Geometry, esp. 57-85. Transl. Carslaw.

Therefore, David Hilbert (1862-1943), in *Grundlagen der Geometrie* (1899; *The Foundations of Geometry*, 1902), actually went so far as to abandon evidence as a criterion of truth. Instead of the fundamental concepts of Euclid's geometry, such as point, straight line and plane, he uses corresponding variables, "x", "y" and "z", which are not explained in terms of their content, but which can in principle be interpreted at will. Geometrical axioms, then, no longer need to be evident, but are conventions arbitrarily fixed between these variables. They are merely syntactical characters without any content. However, they must be consistent and independent of each other. From an inconsistent system of axioms, one would be able to derive anything one wished.

According to a law of logic – if (p and not p) then q – any conclusion q may follow from a logical contradiction. The small letters p and q are propositional variables standing for any concrete proposition. For example, we could substitute: If the parallel axiom is true (p) and not true (not p), then Hilbert is an unhappy man (q). But Hilbert did not want to prove this proposition in *Foundations of Geometry.* Axioms that depend on each other would be derivable from each other and would no longer be axioms.

Hilbert separates the logical and formal element from the concrete, and declares consistency to be the criterion of truth and (logical) existence. Thus, he writes to Frege: "If the arbitrarily given axioms do not contradict one another with all their consequences, then they are true and the things defined by them exist. This for me is the criterion of truth and existence."[23] Only in a second step does he assign a semantic to the basic terms and axioms, for example, the meaning of "point", "straight line"

[23] Frege, Letters, 411. Transl. Geach and Black.

or “plane”, or the meaning of the Euclidean axioms, albeit without the parallel axiom.

The dismissal of evidence as a criterion of truth has an important consequence. With evidence in use, it still seemed possible to claim that an axiom was evident in the sense of being true “in itself”. Now it is no longer possible to maintain that an axiom is true “in itself”, but only that it is true within the language community that accepts this particular axiom. Likewise, a proposition is true only within the language of the system of axioms concerned. Thus, the universal validity of the truth of axioms is restricted to the language community, for example, of the mathematicians who share these definitions and the semantics accompanying them.

But we may go somewhat further. Axioms need not be arbitrary constructs. As soon as we allocate a semantic to these syntactical signs, and it is accepted by a language community, the constructs in question come to represent the semantic rules of that language community. And as soon as these rules have stabilised, they become the semantic *institutions* of the language community. Therefore, in my view, the criterion of the truth of axioms need be neither mere evidence nor a consistent definition; it may also be the social fact of their stabilised semantic acceptance.

Such a language community can be very small, as it is, for example, in the case of the non-Euclidean geometries. Here it comprises those mathematicians who construct and teach such geometries. It can be larger, as it is, for example, in the case of Euclidean geometry. Here it consists of all those who accept the axioms of Euclid, including the parallel axiom. It can be even larger, as in the case of the first Euclidean axiom: “Things which equal the same thing also equal one another.” This axiom is also called the axiom of the transitivity of equality: If a equals b, and c equals b, then a also equals c. It is a view shared by most people, except perhaps by lunatics and philosophers – and denied by the latter only when they are philosophising.

The same applies to the metalogical axioms of identity and of non-contradiction. The axiom of identity can be expressed thus: Everything is what it is. According to this axiom, "no entity" is "without identity".[24] The axiom of non-contradiction can be stated as follows: No thing is at the same time and in the same respect another thing. We can combine both axioms and say, with Joseph Butler (1692-1752): "Every thing is what it is, and not [at the same time and in the same respect] another thing."[25]

This is the ontological formulation of the axioms of identity and of non-contradiction. The ontological formulation of the latter axiom goes back to Aristotle: "… the same attribute cannot at the same time belong and not belong to the same subject in the same respect,…".[26] In the ontological formulation, it is necessary to add the temporal qualification "at the same time".

In the logic of the modern age, these axioms have also been called laws of thought and formulated without temporal qualifications. The axiom of identity has been expressed this way: "A equals A." The axiom of non-contradiction hat been stated this way: "A does not equal non-A." If for "equals" we use the sign "=" and for "does not equal" the sign "≠", they will read: "A = A" and "A ≠ non-A." This is the psychological formulation of the axioms of identity and non-contradiction.

But modern logic in its mathematical shape, founded by Frege, no longer talks about laws of thought. It wanted to shed the subjective element and the "unhealthy psychological fast" or

[24] Quine, Ontological Relativity, Chapter 1, 23.
[25] Butler, Sermons, Preface, § 33, 25.
[26] Aristotle, Metaphysics, Book 4, Chapter 4, 1005b19-20. Trans. Ross.

psychological burden attached to our opinions, ideas, judgments and inferences, and to penetrate to objective truth. Moreover, these axioms do not so much *de*scribe how we really think, but rather *pre*scribe how we ought to think. We can, of course, also think illogically.

However, logic is not the science of the most general laws taken to be true, but, according to an apt definition of Frege, “the science of the most general laws of being true”.[27] From “the laws of being true there follow the laws about asserting, thinking, judging, inferring”.[28] That is why logic can also be defined as the general science of inference.

Objectively, the propositions or sentences in which we express our thinking are available to anybody’s perception. The two metalogical axioms are now regarded as propositions. They can be formulated in various ways, the metalogical axiom of identity, for example, as “p is identical to p”, and the metalogical axiom of non-contradiction as “not valid: p and not p”.

If we were to substitute a concrete proposition for the propositional variable p, we would each time obtain the same truth value for these laws, that is, the truth value true. That is why these laws are also called tautologies. Tautologies (from Greek *tautologeín*: to repeat what was said) say the same thing twice. In propositional logic, therefore, tautologies are forms of sentences in which every substitution of a concrete sentence for a propositional variable will result in the same truth value, that is, the truth.

Let us, for example, substitute the concrete proposition “It’s raining” for the propositional variable p. Then the metalogical

[27] Frege, Logic, 139. Transl. Long and White.
[28] Frege, Thought, 342. Transl. Geach and Stoothoff with small modifications by Ferber.

axiom of identity will be "'It's raining' is identical to 'It's raining'." The identity of the two propositions here means that both are either true or false. It does not mean that the first proposition is true and the second false, or that the second is true and the first false. Both propositions have the same, or identical, truth value. That is why we also talk about the equivalence of the two propositions, and may say: "'It's raining' is equivalent to 'It's raining'." If we choose to render the phrase "is equivalent to" by the sign for equivalence "≡" – three parallel lines, in contrast to the two lines meaning equality – it will read: "'It's raining' ≡ 'It's raining'" or, more generally, "p ≡ p". This equivalence can also be expressed in terms of a reciprocal conditional relationship: If the first proposition is true, the second will also be true; if the first proposition is false, the second will also be false and vice versa.

In the case of the metalogical axiom of non-contradiction, the following substitution occurs: "Not valid: 'It's raining' and 'It's not raining'". The propositions "It's raining" and "It's not raining" cannot be both true and false (at the same time and in the same place). Rather than reciprocally determining each other, they reciprocally exclude each other. If "It's raining" is true, then "It's not raining" is false. If "It's not raining" is false, then "It's raining" is true. But the law of non-contradiction is always true if we add the necessary conditions, for example, that it refers to events in the same place and at the same time.

We may call these axioms metalogical truths because they are present as presuppositions not only in Euclid's geometrical axioms, but in the axioms of any special system of logic.[29] For example, the two metalogical axioms mentioned above are pre-

[29] To my knowledge, the term "metalogical truths" for these axioms was introduced by Schopenhauer, Fourfold Root, § 33, 108. Transl. Hillebrand.

supposed in the first axiom of *Principia Mathematica* (1910-1913), the logical system created by Alfred North Whitehead (1861-1947) and Bertrand Russell (1872-1970): "1.1 Anything implied by a true elementary proposition is true."[30]

This axiom means that true premises result in true conclusions. Let us take the following conditional propositions as an example: "If it rains the road gets wet." Let us further assume that "it rains" is an elementary proposition. Then this principle means that if the premise "it rains" is true, then so is the conclusion "the road gets wet". Likewise, the validity of a deductive argument presupposes that the affirmation of the premises and the negation of the conclusion result in a logical contradiction, while the affirmation of the premises and the affirmation of the conclusion does not.

Of course, a radical sceptic could also deny the metalogical axioms of identity and non-contradiction. Even though there has hardly ever been such a sceptic, his position can be formulated as a hypothesis. In order to negate the metalogical axioms, he would first have to affirm them. If he said, "The axiom of identity is not true", he would be presupposing the following proposition: "'The axiom of identity is not true' is identical to 'the axiom of identity is not true'." But if he substituted the word "equivalent" for "identical", he would be assuming the proposition: "'The axiom of identity is not true' is equivalent to the proposition 'the axiom of identity is not true'." Here the word "equivalent" is only a different word for "identical" that expresses the identity of the truth value. In both cases, the radical sceptic would still presuppose the axiom of identity in order to negate it.

[30] Whitehead/Russell, PM, Part I, Section A, 94.

But let us further imagine him saying: “The axiom of non-contradiction is not true.” In that case, he would presuppose that the sentence “The axiom of non-contradiction is not true” and its negation, “The axiom of non-contradiction is true”, are not true simultaneously. But by this presupposition, he will be affirming the axiom of non-contradiction. If he affirms the axiom of non-contradiction, he does not negate it. But if he does not negate it, even the radical sceptic can no longer advocate the negation of the law of non-contradiction. He cannot advocate negating it, because in order to advocate negating it, he has to affirm it.

If the radical sceptic could no longer advocate his own theoretical position, he would have to resign from any verbal debate with his opponent and be condemned to silence. Since he no longer advocated any theoretical position, he would indeed be irrefutable, albeit not because he was advocating an irrefutable theoretical position, but because he was no longer saying – and able to say – anything definite, for any proposition he made would also mean its opposite. At best, he would be able to express his position in body language, for example, by shaking his head doubtfully if somebody stated the axiom of identity or non-contradiction. But even this doubtful shaking of the head would convey an unclear meaning, as it could express either affirmation or negation.

In contrast, the metalogical axiom of the excluded third – which claims with reference to any sentence p: “p or not p. There is no third” – is not true of every system of axioms in logic and mathematics. It is not true, for example, in the system of Luitzen Egbertus Jan Brouwer (1881-1966). According to Brouwer, mathematical propositions can be considered true or false only if they are provable or refutable by means of a construction. But, when dealing with infinity, we cannot assume that every mathematical sentence will be provable or refutable, with no third possibility between them.

For example, there are perfect numbers and imperfect numbers. A perfect number is a natural number that is equal to the sum of its divisors. Thus, the number 6 is perfect, since 6 = 1+2+3. The number 28 is perfect, since 28 = 1+2+4+7+14. So are 496 and six other even numbers, since their sum is also equal to the sum of their divisors. But so far, no odd number has been proved to be a perfect number. This does not mean that all odd numbers are imperfect. Rather, a sentence such as "All odd numbers are imperfect" is neither provable nor refutable by a construction, since there are infinitely many odd numbers. That is why, according to Brouwer, the metalogical law of the excluded third is not true in propositions about an infinity of numbers.

In my view then, axioms are true neither because they are always evident nor because they are laid down consistently, but because they are institutionalised in a language community. Those who fail to accept them do not belong to that language community. The institutions of a language community are not only laws of being true, *de*scribing what is the case in that community, but they are also rules *pre*scribing what should be taken for truth in that community. Thus, the institutionalist understanding of axioms shows not only why these axioms *are* true in a language community, but also why the members of the language community in question *ought* to follow these axioms.

This institutionalist view of axioms may seem sobering. But if it is true, there can be no absolute justification of the truth of axioms, but only a relative justification by the semantic institutions of the language community concerned. Naturally, these must be consistent and independent of each other. On the basis of this merely relative justification, in my opinion, we can no longer assert that axioms are timeless and true everywhere.